**Unit - I: Nature and Scope of Mathematics**

Meaning and scope of mathematics.

1. Nature, scope and development of Mathematics. Introduction :- Mathematics plays an important role in accelerating the social, economical and technological growth of a nation. It is more so in India, as nation is rapidly moving towards globalization in all aspects. The world of today which leans more and more heavily on science and technology demands more and more mathematical knowledge on the part of its people. So, it is necessary to prepare the child with a strong base of mathematical knowledge to face the challenges of the modern technological society. Etymology:- The term “Mathematics” is derived from two Greek words:

[2.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-2-638.jpg?cb=1542184919) In Hindi, we call Mathematics as “Ganitha”- which means the science of calculations.• According to “New English Dictionary” “Mathematics – in a strict sense – is the abstract science which investigates deductively the conclusions implicit in the elementary conception of spatial and numerical relations” • The dictionary meaning of Mathematics is that, “It is the science of number (or) space” (OR) “The science of measurement, quantity and magnitude” •‘Manthanein’ means ‘learning’ ‘Techne’ means ‘an art (or) technique’ Mathematics means the art of learning related to disciplines (or) facilities. What is Mathematics?

[3.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-3-638.jpg?cb=1542184919)Hence, we can conclude that, Mathematics is a systematized, organized and exact branch of science. Also, Mathematics is the science of quantity, measurement and spatial relations. Mathematics – in words of different authors :- -Kant: “Mathematics is the indispensable instrument of all physical resources.” -C.F.Gauss: “Mathematics is the queen of science and Arithmetic is the queen of all Mathematics.” -Bacon: “Mathematics is the gateway and key to all science.”

[4.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-4-638.jpg?cb=1542184919)-Benjamin Franklin: “What science can there be nobler, more excellent, more useful for men, more admirable, high and demonstrative than that of Mathematics?” -Locke: “Mathematics is a way to settle in the mind a habit of reasoning.” -J.B.Shaw: “Mathematic is engaged, in fact, in the profound study of art and the expression of beauty.” Though there are innumerable definitions of Mathematics, none of them is comprehensive enough to bring out the meaning of Mathematics full. However, each definition throws insight in to one (or) more aspects of Mathematics.

[5.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-5-638.jpg?cb=1542184919)Meaning & Mathematics is not to be considered only as ‘number work’ (or) ‘computation’, but it is more about forming generalizations, seeing relationships and developing logical thinking¬It is a logical study of shape, arrangement and quantity. ¬Mathematic deals with quantitative facts, relationships as well as with problems involving space and form. ¬Mathematics is a systematized, organized and exact branch of science. ¬definitions of Mathematics :- & The National Policy on Education (1986) states, ‘Mathematics should be visualized as the vehicle to train a child to think, reason, analyses and to articulate logically.”¬Mathematics should be shown as a way of thinking, an art (or) form of beauty and as human achievement. ¬reasoning.

[6.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-6-638.jpg?cb=1542184919)It is an exact science and involves high cognitive abilities and powers.

**Nature of Mathematics:**-

The nature of Mathematics can be made explicit by analyzing the chief characteristics of Mathematics.

(i) Mathematics is a science of Discovery: E.E.Biggs states that, “Mathematics is the discovery of relationships and the expression of those relationships in symbolic form – in words, in numbers, in letters, by diagrams (or) by graphs.”¬It provides opportunity for the intellectual gymnastic of the man’s inherent powers. ¬Mathematics helps in solving problems of life that needs numeration and calculation. ¬

[7.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-7-638.jpg?cb=1542184919)According to A.N.Whitehead, “Every child should experience the joy of discovery.” Mathematics gives an easy and early opportunity to make independent discoveries. The children must have opportunities for making their own discoveries of mathematical ideas, but they must also have the practice necessary to achieve accuracy in their calculations. Today it is discovery techniques, which are making spectacular progress. They are being applied in two fields: in pure number relationships and in everyday problems like money, weights and measures.

(ii)Mathematics is an intellectual game: Mathematics can be treated as an intellectual game with its own rules and abstract concepts. From this view points, Mathematics is mainly a matter of puzzles, paradoxes and problem solving – a sort of healthy mental exercise.

(iii)Mathematics deals with the art of drawing conclusions: One of the important functions of the school is to familaries children with a mode of thought which helps them in drawing right conclusions and inferences. According to Benjamin Pierce, “Mathematic is the science that draws necessary conclusions.” In Mathematics, the conclusions are certain and definite. Hence, the learner can check whether (or) not he has drawn the correct conclusions, permit the learner to begin with simple and very easy conclusions, gradually move over to more difficult and complex ones.

[9.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-9-638.jpg?cb=1542184919)(iv)Mathematics is a tool subject: Mathematics has its integrity, its beauty, its structure and many other features that relate to Mathematics as an end in it. However, many conceive Mathematics as a very useful means to other ends, a powerful and incisive tool of wide applicability. In the article “Mathematics & the Teaching Sciences”, John. J. Bowem pointed out that, “Not all students are captivated by the internal consistency of Mathematics and for everyone who makes it a career; there will be dozens to whom it is only an elegant tool.” As Howard. J. Fehr says, “If Mathematics had not been useful, it would long ago have disappeared from our school curriculum as required study.”

[10.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-10-638.jpg?cb=1542184919)(v)Mathematics involves an intuitive method: The first step in the learning of any mathematical subject is the development of intuition. This must come before rules are stated (or) formal operations are introduced. The teacher has to foster intuition in our young children, by following the right strategies of teaching. Intuition when applied to Mathematics involves the concretization of an idea not get stated in the form of some sort of operations (or) examples. Intuition is to anticipate what will happen next and what to do about it. It implies the act of grasping the meaning (or) significance (or) structure of a problem without explicit reliance on the analytic mode of thought. It is a form of mathematical activity which depends on the confidence in the applicability of the process rather than upon the importance of right answers all the time. It is up to the teacher to allow the child to use his natural and intuition way of thinking, by encouraging him to do so and honoring him when he does.

[11.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-11-638.jpg?cb=1542184919)(vi)Mathematics is the science of precision & accuracy: Mathematics is known as an exact science because of its precision. It is perhaps the only subject which can claim certainty of results. In Mathematics, the results are either right (or) wrong, accepted (or) rejected. There is no midway possible between rights and wrong. Mathematic can decide whether (or) not its conclusions are right. Even when there is a new emphasis on approximation, mathematical results can have any degree of accuracy required. It is the teacher’s job to help the students in taking decisions regarding the degree of accuracy which are most appropriate for a measurement (or) calculation.

(vii)Mathematic is the subject of logical sequence: The study of Mathematics begins with few well – known uncomplicated definitions and postulates and proceeds step by step to quite elaborate steps. Mathematics learning always proceeds from simple

[12.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-12-638.jpg?cb=1542184919)to complex and from concrete to abstract. It is a subject in which the dependence on earlier knowledge is particularly great. Algebra depends on Arithmetic, the Calculus depends on Algebra, Dynamic depends on the Calculus, Analytical Geometry depends on Algebra and Elementary Geometry and so on. Thus gradation and sequence can be observed among topics in any selected branch of Mathematics.

(viii)Mathematics requires the application of rules and concept to new situations: The study of Mathematics requires the learners to apply the skill acquired to new situations. The students can always verify the validity of mathematical rules and relationships by applying them to novel situations. Concept and principle become more functional and meaningful only when they are related to actual practical applications. Such a practice will make the learning of Mathematics more meaningful and significant.

[13.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-13-638.jpg?cb=1542184919)(ix)Mathematics deals with generalization and classification: Mathematics provides ample exercise in combining various results under one head, in making schematic arrangements and classifications. When the pupil evolves his own definitions, concept and theorems, he is making generalizations. The generalizations and classification of Mathematics are very simple and obvious in comparison with those of other domains of thought and activity. However, the Mathematics teacher should take care to see that the final generalization into a rule should always be deferred until it is almost spontaneously suggested by the pupils themselves.

(x) Mathematics is an abstract Science: Mathematical concepts are abstract in the sense that they cannot be seen (or) felt in the physical world.

. (xi)Mathematics is study of structures: The dictionary meaning of ‘structure’ is, ‘the formation, arrangement and articulation of parts in anything built up by nature or art’. Therefore, a mathematical structure should be some sort of arrangement, formation (or) result of putting together of parts. For example, we take as the fundamental building units of a structure the members a ,b ,c,…….. of a non empty set S, we hold together these building units by using one or more operations, namely addition or multiplication. A mathematical structure is a mathematical system with one or more explicitly recognized

[17.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-17-638.jpg?cb=1542184919)(xii)Mathematics is a science of logical reasoning: It goes without saying that logic is an important factor in mathematics. It governs the pattern of deductive proof through which mathematics is developed. Of course, logic was used in mathematics centuries ago. According to Russel and Whitehead, “Mathematics is logic during the last few decades there has been great emphasis on the analysis of the logical structure of mathematics as a whole.” Polya suggested that, “Mathematics actually has two faces. One face is a systematic deductive science. The second face of mathematics is in the making it appears

as an experimental, inductive science”.

Reasoning in Mathematics is of two types :

[31.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-31-638.jpg?cb=1542184919)(i) Inductive reasoning (ii) Deductive reasoning

(i)Inductive reasoning: When statements containing mathematical truths are based on general observations and experiences, the reasoning is called “inductive reasoning.” In this type of reasoning, we argue that a particular property is true in a number of cases, so it will be true in all similar cases.

(ii)Deductive reasoning: This type of reasoning is based on certain postulates (or) axioms and in this the statements are products of mind. We try to compare and contrast various statements and then draw our conclusions from such a comparison. Essentials of a deductive reasoning are as follows: (a) Undefined terms (b) Definitions (c) Postulates and axioms.

[32.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-32-638.jpg?cb=1542184919) Set, number, variable,… etc un Algebra. (b) Definitions:- We define many a technical terms in Mathematics only with the help of undefined terms. For example, If A, B, C are three non – collinear points, then triangle ABC is the union of the line segments AB, BC, CA. Thus to define a triangle, we here used the terms collinear, point, union and line segment.♣ point, surface,… etc in case of Geometry

♣(a) Undefined terms:- As in the other branches of science, so also in Mathematics, we come across many terms which cannot be precisely defined. In modern Mathematics, we accept certain undefined terms. For example,

1. [33.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-33-638.jpg?cb=1542184919)( c) Postulates & Axioms:- Early Greeks considered ‘postulates’ as ‘general truths common to all studies’ and ‘axioms’ as ‘the truth relating to the special study at hand’. However, later on modifications were made and ‘postulates’ are considered as ‘permissible constructions’ and ‘axioms’ refer to ‘all other initial assumptions’. In modern Mathematics, the two words ‘postulates’ and ‘axioms’ are used synonymously.
2. [34.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-34-638.jpg?cb=1542184919)Significance of postulates and axioms in teaching of Mathematics:- Postulates:- We know ‘postulates’ are the self – evident truths which are taken for granted without any necessity of proof (or) explanation. These are the first principles from which we deduce mathematical conclusions (or) inferences through a process of reasoning. To prove a theorem, we base our arguments on the previously proved statements. If we go back in this chain, we arrive at the earliest statements, prior to which no proved statements are available. For proving these initial statements, we have to depend upon our postulates, the self – evident truths. Postulates are suppositions, without proof.

[35.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-35-638.jpg?cb=1542184919)Some of the postulates are as follows: (i) A straight line can be drawn from any point to any other point. (ii) A finite straight line can be produced to any length in the line. (iii) A straight line has one and only one middle point. (iv) Two straight lines cannot interest in more than one point. (v) Any angle can by bisected by one and only one line. (vi) We can draw a circle with given centre and passing through a point. (vii) All right angles are equal. (viii) Any geometric figure can be moved without changing its shape and size. (ix) A triangle has interior and exterior angles. (x) Two circles interest at two points. (xi) There is one and only one straight line through a point parallel to given line.

[36.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-36-638.jpg?cb=1542184919)Axioms:- Euclid regarded most other initial assumptions as axioms and called them ‘common notion’. They were accepted as true because of their conformity with common experience and sound judgment. The important notations are given below: (i) Things equal to the same thing, are equal to one another. (ii) The whole is equal to the sum of its parts and is greater than part. (iii) If a > b, b > c, then a > c. (iv) Magnitudes of figures which can be made to coincide with one another are equal.

[37.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-37-638.jpg?cb=1542184919)Teaching Mathematics 2 methods A) Inductive method:- Inductive method is the way of proving any universal truth by showing that, if it is for any particular case, then it is true for the next case in the same serial order. This method takes in to account the process of induction. In this method, students are required not to accept the already discovered formula without knowing how it has been established. They are helped in its discovery by adopting inductive reasoning. Inductive reasoning is a process in which one proceeds from particular to general, from concrete facts to abstract rules and from the special examples to the general formula.

1. [38.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-38-638.jpg?cb=1542184919)Inductive reasoning in our daily life: \* A child observes the rising sun and getting of darkness after setting of the sun. He observes this for a particular day and the next day also, the same thing happens. So, he came to the conclusion that “The sun rises every day and also sets every day.” \* A child eats green apple and feels sour taste. On the next also, he takes another green apple and feels the same. So, he concludes that, “green apples are sour.” \* Scientists also use this method. Only after weighing so many types of matter at so many places, they arrived at the conclusion, “The matter has weight.” So, inductive reasoning is nothing but learning from direct experiences.
2. [39.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-39-638.jpg?cb=1542184919)Use of Inductive reasoning in teaching Mathematics: \* Establishing the formula : (a + b)2 = a2 + b2 + 2ab. Students are asked to find out the values of (a + b)2 , (x + y)2 , (m + n)2 , (p + q)2 … etc by simple multiplication. After doing those multiplications, they can conclude that, (1st term + 2nd term)2 = (1st term)2 + (2nd term)2 + 2(1st term) (2nd term). B) Deductive method:- Deductive method is the one in which first we have to accept the pre – constructed form as well established truth and then we have to check it for particular problems.
3. [40.](https://image.slidesharecdn.com/angelnewnotes-181114084055/95/nature-scopemeaning-and-definition-of-mathematics-pdf-4-40-638.jpg?cb=1542184919)Deductive reasoning in daily life :- \* A child may tell that, he should never eat the green apples because they are sour. Afterwards he may verify this fact by tasting green apples. Use of deductive reasoning in teaching Mathematics:- \* Student may be told about the formula of the area of a rectangle. i.e.) Area = Length x Breadth. Then they are asked to apply it in finding the areas of different rectangles.

A mathematical theorem and its variants—

converse, inverse and contra-positive,

Given an if-then statement "if pp , then qq ," we can create three related statements:

A conditional statement consists of two parts, a hypothesis in the “if” clause and a conclusion in the “then” clause.  For instance, “If it rains, then they cancel school.”   
  *"It rains"*is the hypothesis.  
  *"They cancel school"*is the conclusion.

To form the converse of the conditional statement, interchange the hypothesis and the conclusion.  
      The converse of *"If it rains, then they cancel school"*is *"If they cancel school, then it rains."*

To form the inverse of the conditional statement, take the negation of both the hypothesis and the conclusion.  
      The inverse of *“If it rains, then they cancel school”*is *“If it does not rain, then they do not cancel school.”*

To form the contrapositive of the conditional statement, interchange the hypothesis and the conclusion of the inverse statement.   
      The contrapositive of *"If it rains, then they cancel school"*is *"If they do not cancel school, then it does not rain."*

|  |  |
| --- | --- |
| Statement | If pp , then qq . |
| Converse | If qq , then pp . |
| Inverse | If not pp , then not qq . |
| Contrapositive | If not qq , then not pp . |

For example-

* Inverse of converse is contrapositive.
* Inverse of contrapositive is converse.
* Converse of inverse is contrapositive.
* Converse of contrapositive is inverse.
* Contrapositive of inverse is converse.
* Contrapositive of converse is inverse.
* If today is Sunday, then it is a holiday.
* If 5x – 1 = 9, then x = 2.
* If it rains, then I will stay at home.
* I will dance only if you sing.
* I will go if he stays.
* We leave whenever he comes.
* You will qualify GATE only if you work hard.
* If you are intelligent, then you will pass the exam.

**proofs and types of proofs,**

A **mathematical proof** is an [inferential](https://en.wikipedia.org/wiki/Inference) [argument](https://en.wikipedia.org/wiki/Argument-deduction-proof_distinctions) for a [mathematical statement](https://en.wikipedia.org/wiki/Proposition), showing that the stated assumptions logically guarantee the conclusion. The argument may use other previously established statements, such as [theorems](https://en.wikipedia.org/wiki/Theorems); but every proof can, in principle, be constructed using only certain basic or original assumptions known as [axioms](https://en.wikipedia.org/wiki/Axiom),[[2]](https://en.wikipedia.org/wiki/Mathematical_proof#cite_note-2)[[3]](https://en.wikipedia.org/wiki/Mathematical_proof#cite_note-nutsandbolts-3)[[4]](https://en.wikipedia.org/wiki/Mathematical_proof#cite_note-4) along with the accepted rules of [inference](https://en.wikipedia.org/wiki/Inference). Proofs are examples of exhaustive [deductive reasoning](https://en.wikipedia.org/wiki/Deductive_reasoning) which establish logical certainty, to be distinguished from [empirical](https://en.wikipedia.org/wiki/Empirical_evidence) arguments or non-exhaustive [inductive reasoning](https://en.wikipedia.org/wiki/Inductive_reasoning) which establish "reasonable expectation". Presenting many cases in which the statement holds is not enough for a proof, which must demonstrate that the statement is true in *all* possible cases. An unproven proposition that is believed to be true is known as a [conjecture](https://en.wikipedia.org/wiki/Conjecture), or a hypothesis if frequently used as an assumption for further mathematical work.[[5]](https://en.wikipedia.org/wiki/Mathematical_proof#cite_note-:0-5)

Proofs employ [logic](https://en.wikipedia.org/wiki/Logic) expressed in mathematical symbols, along with [natural language](https://en.wikipedia.org/wiki/Natural_language) which usually admits some ambiguity. In most mathematical literature, proofs are written in terms of rigorous [informal logic](https://en.wikipedia.org/wiki/Informal_logic). Purely [formal proofs](https://en.wikipedia.org/wiki/Formal_proof), written fully in [symbolic language](https://en.wikipedia.org/wiki/Symbolic_language_(mathematics)) without the involvement of natural language, are considered in [proof theory](https://en.wikipedia.org/wiki/Proof_theory). The distinction between [formal and informal proofs](https://en.wikipedia.org/wiki/Proof_theory#Formal_and_informal_proof) has led to much examination of current and historical [mathematical practice](https://en.wikipedia.org/wiki/Mathematical_practice), [quasi-empiricism in mathematics](https://en.wikipedia.org/wiki/Quasi-empiricism_in_mathematics), and so-called [folk mathematics](https://en.wikipedia.org/wiki/Mathematical_folklore), oral traditions in the mainstream mathematical community or in other cultures. The [philosophy of mathematics](https://en.wikipedia.org/wiki/Philosophy_of_mathematics) is concerned with the role of language and logic in proofs, and [mathematics as a language](https://en.wikipedia.org/wiki/Mathematics_as_a_language).

* [3.1Direct proof](https://en.wikipedia.org/wiki/Mathematical_proof#Direct_proof)
* [3.2Proof by mathematical induction](https://en.wikipedia.org/wiki/Mathematical_proof#Proof_by_mathematical_induction)
* [3.3Proof by contraposition](https://en.wikipedia.org/wiki/Mathematical_proof#Proof_by_contraposition)
* [3.4Proof by contradiction](https://en.wikipedia.org/wiki/Mathematical_proof#Proof_by_contradiction)
* [3.5Proof by construction](https://en.wikipedia.org/wiki/Mathematical_proof#Proof_by_construction)
* [3.6Proof by exhaustion](https://en.wikipedia.org/wiki/Mathematical_proof#Proof_by_exhaustion)
* [3.7Probabilistic proof](https://en.wikipedia.org/wiki/Mathematical_proof#Probabilistic_proof)
* [3.8Combinatorial proof](https://en.wikipedia.org/wiki/Mathematical_proof#Combinatorial_proof)
* [3.9Nonconstructive proof](https://en.wikipedia.org/wiki/Mathematical_proof#Nonconstructive_proof)
* [3.10Statistical proofs in pure mathematics](https://en.wikipedia.org/wiki/Mathematical_proof#Statistical_proofs_in_pure_mathematics)
* [3.11Computer-assisted proofs](https://en.wikipedia.org/wiki/Mathematical_proof#Computer-assisted_proofs)

A proof is a logical argument that tries to show that a statement is true. In math, and computer science, a proof has to be well thought out and tested before being accepted. But even then, a proof can be [discovered to have been wrong](https://mathoverflow.net/questions/35468/widely-accepted-mathematical-results-that-were-later-shown-wrong). There are many different ways to go about proving something, we’ll discuss 3 methods: direct proof, proof by contradiction, proof by induction. We’ll talk about what each of these proofs are, when and how they’re used.

Before diving in, we’ll need to explain some terminology.

A **theorem** is a mathematical statement which is proven to be true.

A statement that has been proven true in order to further help in proving another statement is called a **lemma.**

**Direct Proof (Proof by Construction)**

Many theorems state that a specific type or occurrence of an object exists. One method for proving the existence of such an object is to prove that P ⇒ Q (P implies Q). In other words, we would demonstrate how we would build that object to show that it can exist. A proof by construction is just that, we want to prove something by showing how it can come to be. There are only two steps to a direct proof :

1. Assume that P is true.

2. Use P to show that Q must be true.

Let’s take a look at an example.

Theorem: If a and b are consecutive integers, the

sum of *a + b*must be an odd number.

Following the steps we laid out before, we first assume that our theorem is true. We then can say that since *a*and *b* are consecutive integers, *b* is equal to *a + 1*. In that case, *a + b* can be rewritten as *a + a + 1*or *2a + 1*. Therefore, we can say that *a + b = 2k + 1.* We know that any number multiplied by an even number must be even. We also know that if we add 1 to any even number, it becomes odd. Given these, we can say: *a + b = 2k + 1*shows that *a + b*is odd.

**Proof by Contradiction**

A common form of proving a theorem is assuming the theorem is false, and then show that the assumption is false itself, and is therefore a contradiction.

Let’s take a look at a simple example:

Theorem: If n² is even, then n is even.

Given this theorem, let’s assume that n² is even but n is odd. We’re assuming that the theorem is false. As we showed in the previous section, an odd number can be characterized by n = 2k + 1. Using that definition for an odd number we say the following:

n² = (2k + 1)² = 4k²+ 4k + 1 = 2(2k² + 2k) + 1

Or more concisely, *n² = 2(2k² + 2k) + 1*. If we let *m = 2k² + 2k,*we get *n² = 2m + 1*. Using the definition for odd numbers that we mentioned before, we must say that *n²*is odd. In our assumption, we declared *n²*to be even. A contradiction! Since our assumption cannot be, then *n²* must be even, and we’ve proven the original theorem.

**Proof by Induction**

Proof by induction is a more advanced method of proving things, and to be honest, something that took me a while to really grasp. This method is used to show that all elements in an infinite set have a certain property. For example, we may want to prove that 1 + 2 + 3 + … + n = n (n + 1)/2.

In a proof by induction, we generally have 2 parts, a **basis**and the **inductive step.** The basis is the simplest version of the problem, In our case, the basis is,

For n=1, our theorem is true

since, 1 = 1(1 + 1)/2 .

The next part of the proof is the inductive step. The inductive step is the part where to generalize your basis and take it a step further.

Suppose our theorem is true for some n = k ≥ 1, that is:

1 + 2 + 3 + . . . + k = k(k + 1)/2.

Prove that our theorem is true for n = k + 1, meaning:

1 + 2 + 3 + . . . + k + (k + 1) = (k + 1)(k + 2) 2 .

This is the core of the inductive step. We take our theorem, generalize it and take it to the next step. We added in the *(k + 1)*on the left side of the equals sign and we changed the*k*on the right side of the equals sign to *(k + 1)(k + 2).*We finally finish off our proof with

And with that, we’re done. We’ve followed a logical progression from the basis or the base case, to the inductive step, all the way through to the final part of the proof.

# Conclusion

We looked at a few different types of proofs and how they really work. We can use these methods to make logical arguments about the validity of some statement in everyday life, or in the code that we right, or in countless of other situations.

**difference between proof and verification;**

1. **Verify** means **"check"**, used when you need to check some details or whether an argument is true (in for example an already given proof).
2. **Prove** means that you need to show something is true by finding the argument yourself.

In mathematics, a proof is an inferential argument for a mathematical statement. In the argument, other previously established statements, such as theorems, can be used. In principle, a proof can be traced back to self-evident or assumed statements, known as axioms, along with accepted rules of inference.

A proof must demonstrate that a statement is always true (occasionally by listing all possible cases and showing that it holds in each), rather than enumerate many confirmatory cases.

On other side **verify** is all about checking the previously established or driven theorem and formula’s in each and every scenarios, whether it’s true. In other words, we verify a condition for an already given proof.

Deductive nature of mathematics;

History of mathematics with special emphasis on teaching of mathematics,

contribution of Indian mathematicians.

1. Srinivasa Ramanujan

Ramanujan the mathematical genius taught himself math after he dropped out of high school due to his failure in the English subject. He is most famously known for his contribution in analytical theory of numbers, elliptic functions, continued fractions and infinite series. He was also invited to England on his set of 120 theorems that he sent to Cambridge. He further made many mathematical demonstrations in his lifetime, all of which are beyond the scope of this article. He taught a greater valuable lesson, that failure isn’t permanent as he did not let his failure bring him down and continued to teach himself mathematics, which he was passionate about. He has been the inspiration of many mathematicians, not just in India but all over the world.

1. Aryabhata

Aryabhata from ancient history was actually the first person to figure out that earth was spherical and revolved around the sun, thereby discovering the nine planets and calculating the correct number of days in a year were 365. Therefore it wouldn’t be wrong to call him a scientist and an astronomer as well, as far back as the time and mythology suggests.

1. Shakuntala Devi

he most famous female Indian mathematician of all time, Shakuntala Devi, was more commonly known as the human computer. She was so called because of her incredible talent to solve calculations without using any calculator. In Dallas she even competed with a computer to give the cube root of 188138517 faster and she won! She went ahead to compete with UNIVAC the world’s fastest computer to solve the 23rd root of a 201 digit number and she won that too! A woman with outstanding talent and outrageous world records!

1. Narendra Karmarkar

Karmarkar graduated from IIT Bombay in electrical engineering and went ahead to proceed with his studies in the U.S. to gain his PhD. He is best known for his work in inventing polynomial algorithms for linear programming. This fine mind created an overlap between international technology and mathematics to give birth to algorithms.

1. Harish-Chandra

The Indian originated American physicist and mathematician is famously known for infinite dimensional group representation theory. He made various contributions throughout his lifetime and has also be awarded with the cole prize by the American mathematical society.

1. C. P. Ramanujam

Besides his passion in the field of mathematics he also enjoyed spending his leisure time listening to music. This brilliant mind achieved the power of knowledge and worked in the fields of number theory and algebraic geometry. He was eventually elected a fellow of Indian Academy of Sciences.

Brahmagupta

He gave four methods of multiplication and his main contribution was the introduction of zero and the fact that zero (0) stood for ‘nothing’ in the world of mathematics.

13. Bhāskara I  
Born in the district of Mysore, this small town lad grew up to be the shining star. His contributions are mainly his proof of the fact that zero stood for ‘nothing’(the idea initially introduced by Bhramagupta). He made many calculations to prove so; division, permutation and combination theories. He also proved how the earth appears to be flat even though it’s a sphere.

14. Bhaskara II  
Bhaskara II so called to avoid any confusion with the first. His work represented significant mathematical and astronomical knowledge. He is most known for his work in calculus and how it is applied to astronomical problems and computations. Not only did he deal with calculus but had vast knowledge over arithmetic, algebra, mathematics of planets and spheres.

15. Hemachandra  
His most significant contribution in mathematics was his initial version of the Fibonacci sequence. He was not only a mathematician but also a scholar, polymath, poet who wrote on grammar, philosophy and contemporary history. Therefore his contributions are not only restricted to math but over all the various different fields that he had mastered over.

Aesthetic sense in mathematics and beauty

in mathematics.

**Unit - II: Exploring Learners**

Exploring maths creatively:

1. ***Empowers pupils to take ownership of their learning as active learners***

It is no longer about the teaching but rather about the learning. The distinction reflects the important notion that pupils should be actively engaged in the learning process. Pupils are given the opportunity to enquire, investigate and choose from a variety of resources. They can direct the focus of their learning according to their interests and prior knowledge. Pupils’ motivation and expectations increase and so does their confidence in engaging with maths skills and concepts.

1. ***Promotes investigative and problem-solving skills***

Creative mathematics is all about developing problem-solving skills which enables pupils to solve unfamiliar mathematical problems creatively. Pupils realise that there might be more than one possible solution to solving a given situation and learn how to adopt diverse strategies towards problem-solving which best suit their learning styles, capabilities and situation. Pupils are also given the time, space and resources to explore mathematical skills and concepts and can devise their own path to a solution.

1. ***Establishes connections to real life making learning more relevant***

The notion of a classroom has been subject to strong competition with the real world beyond its walls, as well as the instantly accessible virtual world. In today’s information-based and highly globalised society, it is simply absurd to teach without acknowledging real data that is surrounding and bombarding us every second. Teaching and learning should be ever more connected and contextualised in real life circumstances. We cannot have pupils ask; “Why are we learning this?”. The more we establish links between learning and real life, the better can pupils apply their knowledge and skills, and regard the learning as valuable and relevant. Learning tasks should be more based on real life situations, enabling learners to tap into their prior knowledge whilst becoming more engaged with the task at hand. Such examples of realia include; menus, TV schedules, informative websites, transport information, published newsletters, promotional leaflets, sports websites, etc.

1. ***Presents opportunities for collaborative learning and communication***

Creative learning tasks entail the exploration of diverse learning modes which include collaborative group work. Pupils learn to work with other learners who have different learning abilities and together attempt to find a strategy on how to produce something or solve a given task. Throughout this process, the pupils are actively engaged in dialogue. They learn to verbalise their mathematical thinking and to consolidate their use of mathematical vocabulary. Pupils learn to enage in self-assessment and they evaluate their best capabilities, and assign different tasks of the project to specific members of the group in order to reach their final goal. Such tasks allow pupils to develop their social and communication skills, which prepares them for the future. It is the teacher’s responsibility to form functional group clusters by having diverse learners grouped together. Groups should be kept small, so that every student remains engaged and feels important to the rest of the team. One important tip is to assign specific roles within the student’s abilities, and whose duty is necessary in order for the group to reach their final objective or produce their desired outcome.

1. ***Fosters initiative, innovation and creative thinking***

Much focus is being placed upon the terms initiative and creativity, as part of the list of transversal skills required in the 21st century. Society needs citizens who are able to take initiative, who are good decision-makers, problem-solvers and who are able to be creative and think outside the box. Exploring mathematics creatively involves providing open-ended opportunities for our pupils to work collaboratively and to design innovative strategies and solutions to a given situation. This practise allows pupils to foster such important skills which allow them to thrive and to be better equipped for tomorrow’s world.

1. ***Explores maths through technology***

Our children are constantly surrounded by technology especially mobile touchscreen devices. From a very young age they seem to hold an instinctive disposition to interact with screens and to respond to visual cues. This exposure is enforced both in households as well as in other locations outside the home, such as shops, restaurants, shopping centers, etc. Technology has revolutionized the concept of education and has shifted the learning process to one which is more self-directed, creative and also game-based. Exploring maths creatively acknowledges and values the potential and vast resources which technology can provide us. Through technology, pupils learn various skills such as language, creativity, social skills, mathematical thinking, and problem-solving. Technology also provides models and opportunities for pupils to explore their learning through an appealing and relevant medium. One great example of creative maths through technology is the soaring use of coding, which can be carried out either using apps (such as online, mobile or tablet apps) or else through floor robots such as the Pro-Bot. Pupils have the possibility to learn how to programme, design, engage in content creation and problem-solve whilst exploring different skills related to maths such as shape, measure, fractions, angles and position. Coding skills are considered as a language in itself and it forms the basis of logical reasoning. Pupils can be given the opportunity to create their own problem-solving tasks which they can then share with their peers or other pupils from other schools.

1. ***Supports pupils with diverse abilities***

Adopting this approach towards exploring maths in real life, also serves to cater for the different students who are diverse in terms of learning abilities and preferences. Such an approach can be considered inclusive and through the continuous representations of mathematical situations as drawn from real life, pupils will have the chance to explore maths from different perspectives while the learning becomes more appealing. The variety of learning modes, enables most students to participate and to remain engaged on the task.

1. ***Nurtures mathematical thinking and reasoning***

Fundamental to mathematical learning is the ability to think and reason mathematically. It is important to present opportunities whereby the students are able to explore the process of problem-solving through mathematical thinking and reasoning. These situations also enable the learner to become better at communicating their thinking and in finding the appropriate vocabulary to explain their reasoning. Students are also able to observe that there might be more than one possible and reasonable solution to a given problem. Mathematical reasoning can be exemplied verbally, visually or through models.

1. ***Blurs the boundaries among different curricula areas***

One very positive aspect about exploring maths creatively is that it does not only establish a more dynamic relationship among the teacher and the students, but it also establishes links with other learning areas, creating a multi-disciplinary approach to learning. Teachers can collaborate into providing project-based learning scenarios whereby students work in groups and explore a given situation or location and require a myriad number of skills in order to reach their goal or final product. Students learn to establish connections between maths and language, art, history, science, technology, physical education and other aspects of the curricula.

1. ***Heightens understanding and retention***

Hands-on experiences allow students to apply their learnt skills and concepts in practice. Such opportunities provide pupils with a repertoire of experiences which they can recall and allows them to become more confident at applying their knowledge in the future. Through hands-on practice, learners are able to self-assess where they require further support. These activities provide a meaningful context to learning and promotes retention of learnt maths skills and concepts.

Throughout my past years as a maths support teacher, I have had the possibility to visit many primary schools in my country. These visits implied opportunities to collaborate with class teachers and together explore innovative mathematics pedagogies, which appeal to different students and which attracts them towards participating in engaging maths activities. I have gained a lot of insight into the type of quality education which our pupils require not only during their young age but also to increase their participation in tomorrow’s world. We as teachers realise, that we are preparing our learners for a world which is rapidly changing, and for future jobs which are yet to be created. Our teaching has to remain abreast with what technology presents to us and valid to today’s rising generations. It remains, however, our duty to instill in our pupils a sense of love for learning and creating, and to recognise the relevance, interrelatedness and beauty of education in the real world beyond the classroom.

Cultivating learner's sensitivity like intuition,

encouraging learner for probing,

## ****Probing Questions In eLearning****

Probing questions give learners the opportunity to delve into the subject matter and deepen their comprehension of the key concepts and ideas. This form of [inquiry](https://elearningindustry.com/inquiry-based-learning-model)does not have a single correct answer, but allows learners to open up about their personal thoughts, opinions, or assumptions regarding the topic, as all [open-ended questions](https://elearningindustry.com/open-ended-questions-in-elearning-what-elearning-professionals-should-know) do. Probing [**eLearning questions**](https://elearningindustry.com/tags/elearning-questions) can also help to clarify or elaborate on an earlier response, and encourage learners to use their critical thinking skills to determine the motives or underlying causes of the problem.

### ****4 Types of Probing Questions****

1. **Clarification.**A clarification question requires a simple fact-based response. Its sole purpose is to clarify whether the learner understands the concept, and if they need more information in order to fully comprehend the subject matter. “Why do you think that the character in the eLearning scenario performs the process correctly?” is an example of a clarification question, as it gauges whether or not the learner understands the critical steps involved in the described process.
2. **Recommendation.**  
   A recommendation probing question involves a certain degree or [persuasion](https://elearningindustry.com/the-art-of-persuasive-learning-7-tips-for-elearning-professionals). In essence, you are trying to point the learner’s response in a specific direction. An example of a recommendation question would be: “Don’t you think that the narrator of the story should have used a different tool?”
3. **Example.**  
   An example question is ideally suited for responses that may be vague or confusing. For example, if a learner provides an answer that does not contain enough detail, you can ask “Can you give me a specific example?” Their answer will give you a clear indication of whether or not they actually comprehend the subject matter.
4. **Extension.**Extension is a variation of the example question. In the case of extension, learners are encouraged to elaborate upon a response by providing supporting evidence or a reason why they gave the original answer.“Can you tell me why you think the process is flawed?” is an example of an extension question.

raising queries,

appreciating dialogue among peer-group, promoting the student's confidence

## 10 Ways to Build Your Students' Confidence

1. Model confidence.
2. Be prepared to teach.
3. Accept mistakes with grace.
4. Praise and encourage your students.
5. Challenge them academically.
6. Allow your students many opportunities for success.
7. Foster creativity in the classroom.
8. Affirm your students.
9. Give them jobs.
10. Teach them organization skills.

(Carrying out examples from various mathematical content areas such as Number Systems, Geometry, Sets, etc.).

**Unit - III: Aims and Objectives of Teaching School Mathematics**

Need for establishing general objectives for teaching mathematics; Study of the aims and general objectives of teaching mathematics vis-a-vis the objectives of school education; writing specific objectives and teaching points of various content areas in mathematics like Algebra, Geometry,

Trigonometry, etc.

**Unit - IV: School Mathematics Curriculum**

Objectives of curriculum,

principles for designing curriculum,

### Principles of Curriculum Design

1. 1. PRINCIPLES OF CURRICULUM DESIGN FIELD STUDY 4 PREPARED BY: REINALYN CENIZAL PRESENTED BY: TRICIA TRIA, ERICA SARINAS, PRINCESS PEDRO, JOHN LUIS PRULLA FOR PRESENTATION AND SUBMISSION TO: PROF. BETHEL HERNANDEZ
2. [2.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-2-638.jpg?cb=1473040613) Personalization and choice.ϒ Relevance ϒ Coherence ϒ Depth ϒ Progression ϒ Breadth ϒ Challenge and enjoyment ϒPRINCIPLES OF CURRICULUM DESIGN
3. [3.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-3-638.jpg?cb=1473040613)PRINCIPLES OF CURRICULUM DESIGN Why do you think is it important that there are principles in designing a curriculum?
4. [4.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-4-638.jpg?cb=1473040613) Children and young people should find their learning challenging, engaging and motivating. The curriculum should encourage high aspirations and ambitions for all.ϒCHALLENGE AND ENJOYMENT
5. [5.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-5-638.jpg?cb=1473040613)At all stages, learners of all aptitudes and abilities should experience an appropriate level of challenge, to enable each individual to achieve his or her potential.ϒCHALLENGE AND ENJOYMENT
6. [6.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-6-638.jpg?cb=1473040613)They should be active in their learning and have opportunities to develop and demonstrate their creativity. There should be support to enable children and young people to sustain their effort.ϒCHALLENGE AND ENJOYMENT
7. [7.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-7-638.jpg?cb=1473040613)All children and young people should have the opportunities for a broad range of experiences.ϒBREADTH
8. [8.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-8-638.jpg?cb=1473040613)Their learning should be planned and organized so that they will learn and develop through a variety of texts within both the classroom and other aspects of school life.ϒBREADTH
9. [9.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-9-638.jpg?cb=1473040613)Children and you people should experience continuous progression in their learning from 3 to 18 years. Each stage should build upon earlier knowledge and achievements.ϒPROGRESSION
10. [10.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-10-638.jpg?cb=1473040613)Children and young people should be able to progress at a rate which is meets their individual needs and aptitudes.ϒPROGRESSION
11. [11.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-11-638.jpg?cb=1473040613)There should be opportunities for children and young people to develop their full capacity for different types of thinking and learning, exploring and achieving more advanced levels of understanding.ϒDEPTH
12. [12.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-12-638.jpg?cb=1473040613)Children and young people’s learning activities should combine to form a coherent experience. There should be clear links between different aspects of learning.ϒCOHERENCE
13. [13.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-13-638.jpg?cb=1473040613)Such links should be discussed with children and young people in order to bring different strands of learning together.ϒCOHERENCE
14. [14.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-14-638.jpg?cb=1473040613)Children and young people should understand the purpose of their learning and related activities. They should see the value of what they are learning and its relevance to there lives, present and future.ϒRELEVANCE
15. [15.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-15-638.jpg?cb=1473040613)The learning planned for children and young people should respond to their individual needs and support particular aptitudes and talents.ϒPERSONALIZATION AND CHOICE
16. [16.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-16-638.jpg?cb=1473040613)It should provide opportunities for exercising responsible personal choice.ϒPERSONALIZATION AND CHOICE
17. [17.](https://image.slidesharecdn.com/principlesofcurriculumdesign-fs4report-160905015544/95/principles-of-curriculum-design-17-638.jpg?cb=1473040613) Once children and young people have achieved suitable levels of attainment across a wide range of areas of learning, the choice should become as open as possible.ϒPERSONALIZATION AND CHOICE

designing curriculum at different stages of schooling.

Some highlights of curriculum like vision of school mathematics,

main goal of mathematics education,

core areas of concern in school mathematics,

curricular choices at different stages of school mathematics education,

construction of syllabi in various disciplines of mathematics,

for example, Algebra, Geometry, etc.;

Pedagogical analysis of various topics in mathematics at various level of schooling- Arithmetic (Development of Number Systems),Algebra, Trigonometry, Statistics and Probability, etc.

**Unit - V: Approaches and Strategies in Teaching and Learning of Mathematical**

**Concepts**

Nature of concepts, concept formation and concept assimilation,

1. Concept is the process of discrimination of the common features and relations in the world of events, things and persons – Hammerton. 2ϖ Concepts are those thoughts which mention things, incidents, qualities, etc. – Woodworth. ϖ Concept is a process with represents the similarities in otherwise diverse objects, situations or events – Munn. ϖ Concept is a process of representing a common property of objects or events – Morgan. ϖDefinitions
2. [3.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-3-728.jpg?cb=1345231926)Meaning of Concept Formation:•

A concept is the sum total of what we know about the object.• It refers to a generalized idea about the objects/persons/ events. It stands for a general class and not for a particular object/person/event. It is a common name given on the basis of similarities or commonness found in different objects/persons/events. There are concepts of objects such as cat, tree, chair etc., concepts of persons such as mother, Indian, Negro etc., and concepts of qualities such as honesty, goodness, obedience.• It is a mental disposition that helps in understanding the meaning of the objects or perceived earlier.• In one sense, it is general mental image of the objects / persons /events experienced or perceived earlier. 3

1. [4.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-4-728.jpg?cb=1345231926) Concepts are very useful in recognizing, naming and identifying the objects / persons / events. 4ϖ Concept is the process of discrimination of the common features.ϖ

Concert formation is the association of certain stimuli and responses.ϖ Concept can be formed without the use of language.ϖ Concept is a part of thought process.ϖ The concept is not common for all, different persons may have different concept about the same object / events.ϖ A concept is not static, it is always changing. For example first a child considers even the walls and doors to be living things. Then it understands they are nonliving, considers cars, buses and running objects to be alive. Later, it learns that only animals and plants are living.ϖ

Nature of Concept:

1. [5.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-5-728.jpg?cb=1345231926)Types of Concept Formation
2. 1. Direct Experience: It is the first type of concept formation, in which the learner develops concept through direct experience with the particular objects / persons / events. It is developed during from the early childhood onwards. For example, the concept about cow.
3. 2. Indirect Experience: Here the learner develops concept through pictures, photos and reading descriptions, hearing from other. For example, the concept about Kangaroo.
4. Faulty Concepts: The concepts or the general ideas we have about the objects, persons or events, are not always adequate and accurate. Small children have so many concepts that are quite erroneous and inadequate. For example, one’s anxiety over the crossing of his way by a cat or one’s feeling of hatred towards the person belonging to other caste or religion is the result of faulty concepts. Faulty concepts should not be allowed to develop in children. 5
5. [6.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-6-728.jpg?cb=1345231926)Process of Concept Formation:The process of concept formation has three important phases.
6. 1. Perception: Experiences or learning in any form is the starting point of the process of concept formation. Our perceptions or imaginary experiences, formal or informal learning, provide opportunities for getting mental images of the objects, persons or events.

2. Abstraction: The mind analyses the perceived images and synthesizes what is common to all, neglecting what is particular. This process of observing similarities and commonness is named as abstraction.

3. Generalization: After making such observation in the form of abstraction for a numbers of times the child is able to generalize or form a general idea about the common properties of some objects or events. On account of this generalization, he will develop a concept about these things or events. 6

1. [7.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-7-728.jpg?cb=1345231926) In this way he compares and contrasts the similarities or dissimilarities of his mental images related to all perceived cows. In spite of the differences in colour, appearance etc., they are found to possess so many common properties or characteristics. 7∝ Later on, when he perceives a white or red cow he does not at once, call it a cow. He again makes an enquiry and comes to that these are cows. He tries to compare the particular mental image the idea of the previously perceived cow with the images he is having, at present, by perceiving white and red cows.∝ For example, the child perceives a black cow at the first time and is told that it is a cow, he tries to form an idea about it. In the beginning the idea is very particular in nature.∝Concept Formation:
2. [8.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-8-728.jpg?cb=1345231926)Piaget’s Cognitive Development 8
3. [9.](https://image.slidesharecdn.com/conceptformation-120817193126-phpapp01/95/concept-formation-9-728.jpg?cb=1345231926)LOGO

Assimilation refers to a part of the adaptation process initially proposed by [Jean Piaget](https://www.verywellmind.com/jean-piaget-biography-1896-1980-2795549). Through assimilation, we take in new information or experiences and incorporate them into our existing ideas. The process is somewhat subjective because we tend to modify experience or information to fit in with our pre-existing beliefs.

Assimilation plays an important role in how we learn about the world around us. In early childhood, children are constantly assimilating new information and experiences into their existing knowledge about the world. However, this process does not end with childhood. As people encounter new things and interpret these experiences, they make both small and large adjustments to their existing ideas about the world around them.

Let's take a closer look at assimilation and the role it plays in the learning process.

## How Does Assimilation Work?

Piaget believed that there are two basic ways that we can adapt to new experiences and information. Assimilation is the easiest method because it does not require a great deal of adjustment. Through this process, we add new information to our existing knowledge base, sometimes reinterpreting these new experiences so that they will fit in with previously existing information.

In assimilation, children make sense of the world by applying what they already know. It involves fitting reality and what they experience into their current cognitive structure. A child's understanding of how the world works, therefore, filters and influences how they interpret the reality.

For example, let's imagine that your neighbors have a daughter who you have always known to be sweet, polite and kind. One day, you glance out your window and see the girl throwing a snowball at your car. It seems out of character and rather rude, not something you would expect from this girl.

How do you interpret this new information? If you use the process of assimilation, you might dismiss the girl's behavior, believing that maybe it's something she witnessed a classmate doing and that she does not mean it to be impolite. You're not revising your opinion of the girl, you are simply adding new information to your existing knowledge. She's still a kind child, but now you know that she also has a mischievous side to her [personality](https://www.verywellmind.com/what-is-personality-2795416).

If you were to utilize the second method of adaptation described by Piaget, the young girl's behavior might cause you to reevaluate your opinion of her. This process is what Piaget referred to as [accommodation](https://www.verywellmind.com/what-is-accommodation-2795218), in which old ideas are changed or even replaced based on new information.

Assimilation and accommodation both work in tandem as part of the learning process. Some information is simply incorporated into our existing schemas through the process of assimilation while other information leads to the development of new schemas or total transformations of existing ideas through the process of accommodation.

[What Role Do Schemas Play in the Learning Process?](https://www.verywellmind.com/what-is-a-schema-2795873)

## More Examples

* A college student learning how to use a new computer program
* A sees a new type of dog that he's never seen before and he immediately points to the animal and says, "Dog!"
* A chef learning a new cooking technique
* A computer programmer learning a new programming language

In each of these examples, the individual is adding information to their existing schema. Remember, if new experiences cause the person to alter or completely change their existing beliefs, then it is known as accommodation.

Moves in teaching a conceptdefining,

stating necessary and/or sufficient condition, giving examples accompanied by a

reason. Comparing and contrasting; Giving counter examples; Non-examples; Planning and

implementation of strategies in teaching a concept like teaching of algebra, geometry,

trigonometry, mensuration, etc.;

Difference between teaching of mathematics and teaching of science

Science and mathematics are completely independent activities.  
  
Science is discovery. A scientist axiomatizes statements using evidence. "This apple is red" becomes a scientific fact with an event or experiment that documents the reality that backs it. Scientists then build statements that predict facts without having to directly observe them. "All apples are red" would be an example of a predictive statement, and hence a theory. A green apple would easily prove it wrong so "apples fall from trees" would be a better theory.  
  
Mathematics is computation. A mathematician solves problems and derives proofs by applying mathematical axioms to mathematical statements. Truth can only be translated, and never generated. Both sides of any equation are equally true. But this allows a mathematician to juggle the truth without ever breaking it. Mathematics describes the various shapes of any given truth.  
  
When these two meet, great things happen.  
  
When scientific statements are translated into mathematical statements, we can apply mathematics to solve scientific problems. Mathematics helps illustrate the truth we've discovered, and lets us pursue all its logical permutations. Mathematics then allows us to incorporate all of our scientific knowledge into one universal system of symbols and numbers. What this universal system describes is a scientific reality that contains everything about the universe we know so far, to which we all have immediate access. The permanence upheld by this paradigm comprises the basis for the modern common sense intuition of reality.  
  
Science and mathematics go well together. It's great to be able to compute with what we discover. Understanding leads to engineering, and has landed us on the moon. And it's great to discover what we've computed. Predictions are only valuable when validated, and observing the Higgs boson 50 years after we predicted it demonstrates the power of science with mathematics